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Mach bands are phase dependent

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The prevailing explanation of Mach bands, the paradoxical bands of light and dark seen where luminance gradients meet plateaux, is that they are due to lateral inhibition in the visual system[1–4]. This explanation equates Mach bands with distortions in a processed luminance distribution due to selective attenuation of low frequency components. But square waveforms exhibit no Mach bands[2,5,6], although they should also be distorted after processing. Measurements of the contrast required to see Mach bands in trapezoidal waveforms and manipulations of their spectra lead us to conclude that phase relationships between Fourier components are important to the structure we perceive. A model based on the odd and even symmetry of visual receptive fields explains our results.

Mach originally[7] used rotating discs and drums to study the bright and dark bands that appear where ramps of luminance meet plateaux as, for example, at the borders of the penumbra of a shadow. He later devised optical methods to cast shadows[8]. For the studies reported here we used gratings formed by computer on an oscilloscope which were variable in luminance profile and contrast. Fig. 1 shows four trapezoids, including the two extreme cases of a square wave ($t = 0$) and the triangular wave ($t = 0.5$), where t is the ratio of the width of each ramp to the period. Classical Mach bands are seen on the trapezoids (Fig. 1*b*, 1*c*); bright bands are visible at the top of each ramp and dark bands at the bottom. Note also that the ramps do not appear to have a uniform brightness gradient and the plateaux do not appear uniformly dark and light. The triangular wave also exhibits sharp bands at the peaks and troughs of the waveform, but the square wave does not.

The Fourier transforms of the trapezoid waveforms (calculated from the point where the waveform crosses zero) are also shown in Fig. 1. The components are arranged in blocks alternatively positively and negatively weighted (that is, differing in phase by 180°). The number of components in each block varies inversely with t (see formula in the legend), being one for the triangular wave, and infinite for the square wave. The observations that Mach bands are not seen on square waves[2,6], which have no out-of-phase components, or on trapezoids with very steep ramps[5], where the out-of-phase components are all of high spatial frequency, led us to suspect that phase may hold the key to Mach bands.

We isolated residual waveforms from trapezoids by removing the first block of positive harmonics (see Fig. 1*f*) and measured the contrast necessary to see them as a function of spatial frequency (inverse of their period, T in equation (1), Fig. 1, legend). The results are shown by the filled circles in Fig. 2. We then measured the minimum contrast at which Mach bands could be seen in the original trapezoidal waveforms (open triangles in Fig. 2). The two curves follow one another closely as spatial frequency is varied. Interestingly, observers spontaneously remarked that the two tasks, setting thresholds for seeing Mach bands and for detecting residual waveforms, seemed to make common demands on them, as if the two tasks

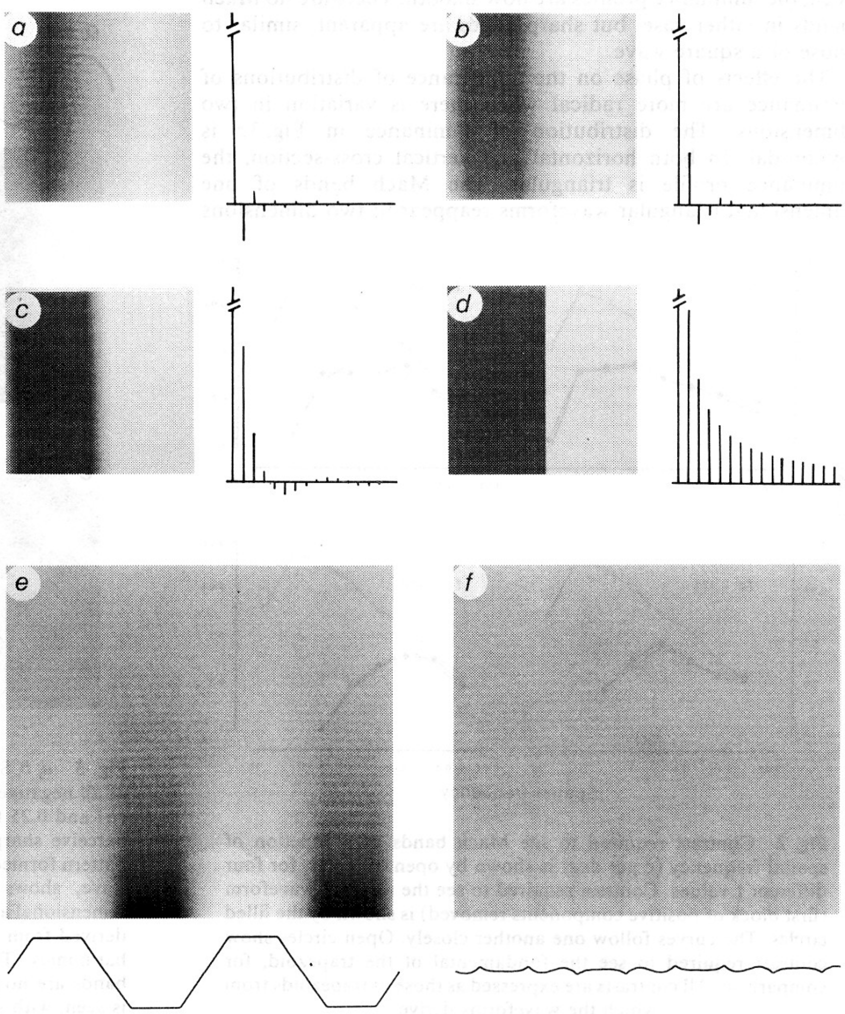

Fig. 1 *a–d*, Examples of four types of trapezoids. The shape of the waveform is determined by the parameter t (the ratio of the ramp width to the period), which is 0.5 in *a* (triangular wave), 0.25 in *b*, 0.125 in *c*, and 0 in *d* (square wave). Mach bands are clearly visible in all except the square wave. The Fourier expansions of the patterns are given by

$$F(x) = \sum_{k=0,\infty} [(4A/t\pi^2) \cdot (\sin(\pi t(2k+1))/(2k+1)^2)] \cdot \sin(2\pi(2k+1)x/T), \qquad (1)$$

where A is the amplitude and T is the period. The spectra are depicted schematically adjacent to each waveform (the first harmonic terms are not to scale). Only the odd terms are non-zero, and they fall into alternatively positively and negatively weighted blocks whose size varies inversely with t. *e, f*, At left a trapezoid waveform ($t = 0.25$), decreasing in contrast exponentially upwards. At right the residual waveform, from which the first block of positively weighted harmonics (the first and the third) has been removed, also decreasing in contrast exponentially upwards. For various distances (spatial frequencies), note the point at which the Mach bands disappear from the trapezoid, and also the point at which the residual waveform falls below detection threshold. The two thresholds should be similar. By fixating some distance below the two panels that Mach bands disappear in the periphery as contrast decreases well before the residual ceases to be visible.

were almost identical. Readers can confirm some of the results for themselves by observing the contrast at which the Mach bands of Fig. 1*e* and the pattern of Fig. 1*f* disappear at various viewing distances.

This result suggests that Mach bands are visible only when the out-of-phase Fourier components (together with all higher harmonics) reach their independent threshold. To test this more directly, we attenuated the higher frequency components by applying a low-pass gaussian filter to various trapezoidal waveforms and decreased the space constant of the gaussian filter until the Mach bands disappeared. We then applied the same filter to the residual waveforms. For all the trapezoids we investigated, the low-pass filter which removed the Mach bands attenuated the appropriate residual waveform to a contrast just below its detection threshold.

It is known that phase discrimination deteriorates in the periphery[9,10]. We measured the contrast required to see Mach bands in various trapezoids and the contrast required to see their residuals as a function of retinal eccentricity. As eccentricity increased the contrast required to see Mach bands increased much more rapidly than that required to see the residual. Indeed, Mach bands could not be seen at all at eccentricities greater than three degress (as previously observed by Mach[8]), whereas the residual was still clearly visible. Phase sensitivity is indeed poor in the periphery; when viewed peripherally, trapezoids become unstable and alternate in appearance between square and triangular waves.

As a final confirmation of the importance of phase, we created synthetic waveforms by inverting the sign of all negatively weighted components of the triangular wave ($t=0.5$) and a trapezoid ($t=0.25$), to make 'all positive' waveforms, having the same amplitude spectra as the parent trapezoids (Fig. 3). As can be seen, the luminance profiles are now smooth. There are no Mach bands in either case, but sharp edges are apparent, similar to those of a square wave.

The effects of phase on the appearance of distributions of luminance are more radical when there is variation in two dimensions. The distribution of luminance in Fig. 3*c* is pyramidal. In both horizontal and vertical cross-section, the luminance profile is triangular. The Mach bands of one dimensional triangular waveforms reappear in two dimensions as star-like patterns where the two dimensional pattern forms points of high and low luminance. Adjacent is the same pattern with all phases set positive. The structure is now like that of a checkerboard, with white and black squares clearly separated by sharp vertical and horizontal borders.

The results of this study suggest that Mach bands are different from edges, but that both depend on phase relationships. When all negative harmonics (180° out-of-phase) are rendered invisible (by lowering their contrast, increasing their spatial frequency, or smooth digital filtering), Mach bands disappear. When patterns are displayed peripherally (where the visual system is poor at resolving phase[9,10]) the bands also disappear. Finally, if all negative harmonics of a trapezoid are shifted in phase by 180°, Mach bands disappear, but an edge is seen where there was none before.

As mentioned earlier, all harmonics of a square wave are in zero phase at the point where the waveform crosses zero in a positive direction and 180° where it crosses zero in a negative direction. This is also true for 'all positive' trapezoids in which all out-of-phase components have been shifted in phase by 180°. Elsewhere, the phases of the harmonics advance at different rates, so no other points have strong congruence of phase. Trapezoidal waveforms other than the square wave have blocks of negatively weighted harmonics which break the congruence of phase at the zero-cross point. However, the periodic halt of the phase advance of the harmonics causes the phases to pile up around ±90° at the points where the ramps meet plateaux. A phase grouping around 90° is typical of that produced by delta functions and bars and may be the signal that produces Mach bands.

Why should the visual system react so strongly to phase relationships? A two dimensional basis is required to extract

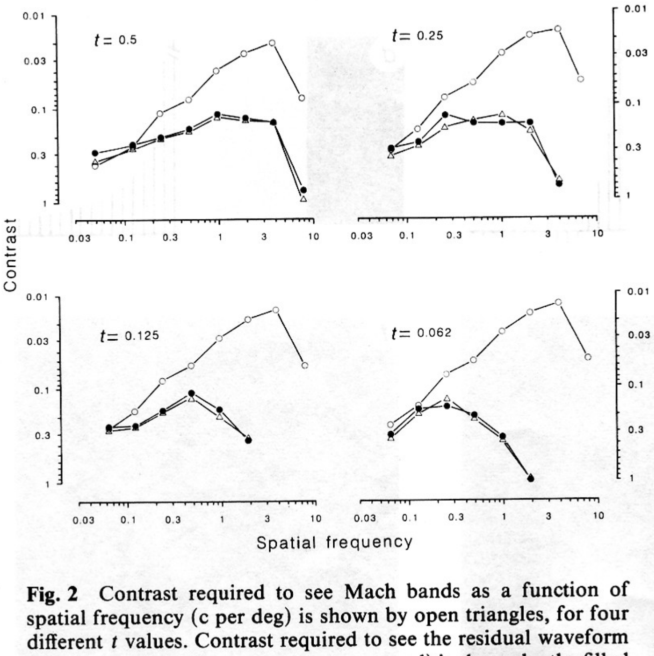

Fig. 2 Contrast required to see Mach bands as a function of spatial frequency (c per deg) is shown by open triangles, for four different *t* values. Contrast required to see the residual waveform (first block of positive components removed) is shown by the filled circles. The curves follow one another closely. Open circles show contrast required to see the fundamental of the trapezoid, for comparison. All contrasts are expressed as those of trapezoids from which the waveforms derive.

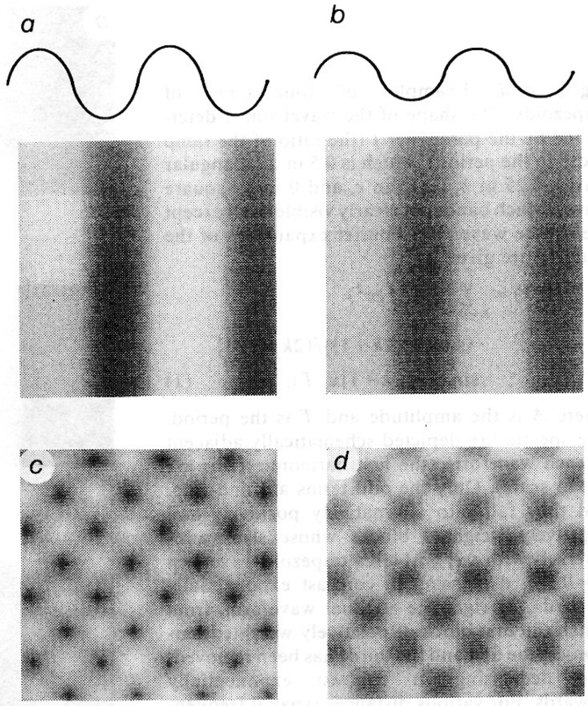

Fig. 3 *a, b*, Synthetic waveforms constructed by inverting the sign of all negatively weighted components of trapezoids of *t* value 0.5 (*a*) and 0.25 (*b*). The waveforms have no discontinuities, yet we perceive sharp edges, with no Mach bands; *c*, two dimensional pattern formed by multiplying a horizontal by a vertical triangular wave, shows white and black star patterns whch are two-dimensional analogues of Mach bands; *d*, two dimensional pattern derived from *c* by inverting the phase of all negatively weighted harmonics. The star patterns reflecting two-dimensional Mach bands are no longer visible. Instead, a checkerboard-like pattern is seen, with sharp borders between the black and white squares.

phase information. The even and odd symmetry of receptive fields of the visual cortex[11] could form such a basis. The receptive fields vary in size, also allowing for coding of an image at different spatial scales[12–15]. The potential ability of such a system to signal edges (odd-symmetric) and lines (even symmetric) has been noted by many investigators[15–18]. We suggest that an efficient means for the visual system to locate bars and edges would be to consider the sum of the squared output of even and odd symmetric filters, which always peaks at points of phase congruence. After the peak had been located the response of the odd and even fields at that point would determine if it were due to an edge or a bar, and give the sign and contrast of the edge or bar. The clustering of phase around 90°, which occurs with trapezoids, should also signal a bar or stripe.

Phase is thought to be encoded by even and odd symmetric detectors in the human visual system[19]. It has further been shown that the sensitivity of the odd, but not the even, symmetric mechanism is reduced in peripheral vision[20]. This would account for the disappearance of Mach bands in the periphery, at contrasts well above those required for the independent detection of out-of-phase harmonics.

Although the generally accepted explanation of Mach bands is that they are due to lateral inhibition, several investigators have noted the problem this presents for square waves and sharp edges[2,5,6,16] and have considered other possibilities. Tolhurst observed that even symmetric receptive fields (which he termed bar detectors) may signal Mach bands[16]. He also speculated that for the square wave the response of odd symmetric fields (edge detectors) may inhibit that of the even symmetric fields (bar detectors), so no stripes are seen (see also ref. 6). Watt and Morgan's[21] general theory of spatial vision also predicts bar signals at the border of a ramp, provided that they are far enough apart. Both these ideas have some similarity to ours, but our results (and simulations to be reported in a fuller paper) suggest that inhibition between edge detectors and bar detectors is unnecessary.

Finally, we note that trapezoids, like square waves and synthetic 'all positive' trapezoids, show zero-crossings at all scales at the mean luminance point. If edges were encoded by alignments of zero-crossings, as has been assumed[22–24], all should have edges at that point. But trapezoids do not. A model seeking phase congruence as the signature of bars and edges, built on adequate basis functions, can without ambiguity, locate edges and bars where we see them.

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