Numerical Estimation in Children With Autism

David Aagten-Murphy, Claudia Attucci, Niki Daniel, Elena Klaric, David Burr, and Elizabeth Pellicano

Number skills are often reported anecdotally and in the mass media as a relative strength for individuals with autism, yet there are remarkably few research studies addressing this issue. This study, therefore, sought to examine autistic children's number estimation skills and whether variation in these skills can explain at least in part strengths and weaknesses in children's mathematical achievement. Thirty-two cognitively able children with autism (range = 8–13 years) and 32 typical children of similar age and ability were administered a standardized test of mathematical achievement and two estimation tasks, one psychophysical nonsymbolic estimation (numerosity discrimination) task and one symbolic estimation (numberline) task. Children with autism performed worse than typical children on the numerosity task, on the numberline task, which required mapping numerical values onto space, and on the test of mathematical achievement. These findings question the widespread belief that mathematical skills are generally enhanced in autism. For both groups of children, variation in performance on the numberline task was also uniquely related to their academic achievement, over and above variation in intellectual ability; better number-to-space mapping skills went hand-in-hand with better arithmetic skills. Future research should further determine the extent and underlying causes of some autistic children's difficulties with regards to number. ***Autism Res** 2015, 00: 000–000*. © 2015 International Society for Autism Research, Wiley Periodicals, Inc.

Keywords: autism; mathematics; number; visual perception

Introduction

Individuals diagnosed with an autism spectrum condition (hereafter, "autism") are often most well known for their difficulties in social communication. It is now well established, however, that they often show an uneven profile of abilities, including a pattern of typical or often superior performance in other domains, particularly in visuospatial processing [e.g., Frith, 1989; Mottron, Dawson, & Soulières, 2009; see Simmons et al., 2009, for review].

Number skills are often reported anecdotally and in the mass media as a relative strength for autistic[1] people. For example, Sacks [1985] reported the extraordinary behavior of two twins with autism, who could determine instantly the correct numbers of matches dispersed on the floor as 111, and subsequently qualify them as "equal to three times thirty-seven." This striking ability, which provided the inspiration for the talents of the lead autistic character in the film, *Rain Man*, was in stark contrast to the twins' cognitive ability ($IQ < 70$). This report echoes many reports of superior calculations abilities in autistic savants [e.g., Cowan & Frith, 2009; Soulières et al., 2010] but whether such superiorities are characteristic of autistic individuals who are not savants is unclear.

One popular theoretical account of autism gives us reason to expect that individuals with autism might be generally talented at mathematics. Baron-Cohen's [2002; Baron-Cohen, Ashwin, Aswin, Tavassoli, & Chakrabarti, 2009] theory proposes that males have an inherent drive to understand rule-based systems (i.e., to "systemize"), which he claims is linked to talent in the fields of mathematics and engineering [Baron-Cohen, Wheelwright, Stott, Bolton, & Goodyer, 1997; although see Jarrold & Routh, 1998]. On this account, autistic individuals are considered to have an "extreme form of the male brain" and, as such, show "hyper-systemizing." One might, therefore, expect individuals with autism also to show enhanced underlying number and mathematical skills.

Despite this widespread belief and the fact that such skills are critical to educational achievement and for achieving independence post full-time education, there are only a handful of studies on this topic [see Chiang & Lin, 2007, for review]. However, with one exception [Iuculano et al., 2014], most studies demonstrate that,

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[1] The term "autistic" is the preferred language of many people on the spectrum (e.g., Sinclair, 1999). In this article, we use this term as well as person-first language to respect the wishes of individuals on the spectrum.

From the Department of Psychology, Ludwig-Maximilians-Universität München, Munich, Germany (D.A.M.); Department of Psychology, University of Florence, Florence, Italy (D.A.M., D.B.); School of Psychology, University of Western Australia, Perth, Australia (D.B., E.P.); Centre for Research in Autism and Education (CRAE), UCL Institute of Education, University College London, UK (C.A., N.D., E.K., E.P.)

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Address for correspondence and reprints: Dr Liz Pellicano, Centre for Research in Autism and Education, Department of Psychology and Human Development, UCL Institute of Education, University College London, 25 Woburn Square, London WC1H 0AA. E-mail: l.pellicano@ioe.ac.uk

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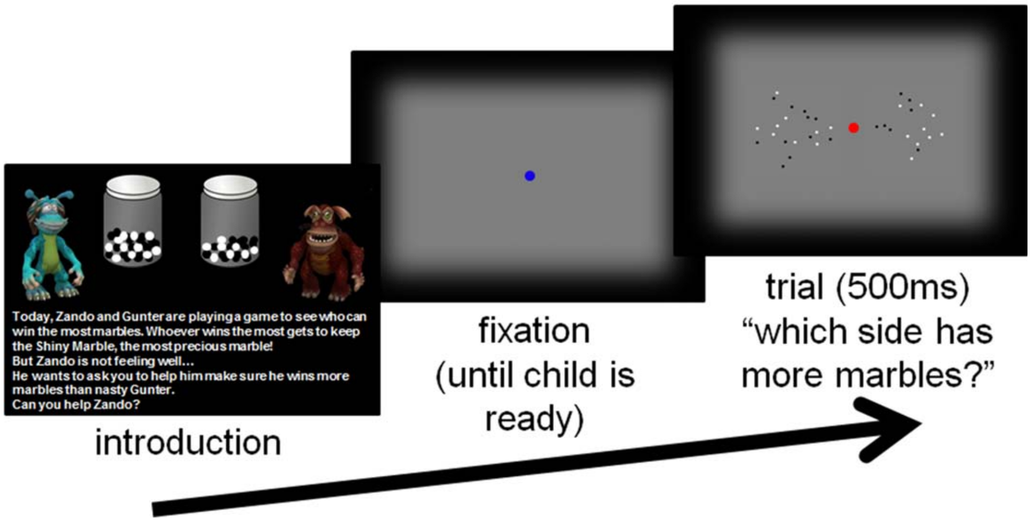

Today, Zando and Gunter are playing a game to see who can win the most marbles. Whoever wins the most gets to keep the Shiny Marble, the most precious marble!
But Zando is not feeling well...
He wants to ask you to help him make sure he wins more marbles than nasty Gunter.
Can you help Zando?

Figure 1. Nonsymbolic estimation: numerosity discrimination task. Children were introduced to two animated characters in a battle to see who can win the most marbles. Children were asked to help the friendly character, Zando (left). They were told they would see two sets of "marbles" briefly and to touch the side of the screen that contains more "marbles."

contrary to Baron-Cohen's [2002] prediction, not all children with autism show strengths in mathematics. Consistent with anecdotal and single-case reports, studies have identified a sizeable proportion of young people with autism [up to 16% of $n = 100$; Jones et al., 2009] with *exceptional* arithmetic processing relative to their intellectual functioning, or "hypercalculia." Yet between 6% [Jones et al., 2009] and 22% [Mayes & Calhoun, 2003a] of children and adolescents with autism were reported to struggle with calculation and mental arithmetic to an extent that their maths difficulties were incommensurate with intellectual functioning, indicative of a specific disability in arithmetic, or developmental dyscalculia.

The amount of variability in arithmetic achievement in autism is striking. Yet as informative as these standardized tests are, they are unable to provide a detailed and comprehensive analysis of *why* such achievement is so variable. Here, we investigated autistic children's numerical estimation skills to determine whether individual differences in these skills might be one potential source of the variability in formal arithmetic skills.

Researchers typically distinguish between two number systems operating over different numerical ranges, both of which are believed to contribute to children's mathematical achievement [Feigenson, Dehaene, & Spelke, 2004]. For relatively few items (<4) a "subitizing" system enables a rapid, *exact* representation of numerosity while an *approximate* system is utilized to estimate magnitudes when more items are present [see Burr & Ross, 2008].

To our knowledge, only three studies have investigated these numerical estimation skills in autism, both targeting the exact number system. Jarrold and Russell [1997] found that autistic children showed less benefit than comparison children in counting dot stimuli presented in canonical (dots on dice) than noncanonical (distributed randomly) form, and used a less efficient dot-by-dot counting strategy. Gagnon, Mottron, Bherer, and Joanette [2004] showed that, when asked to judge numerosities between 2 and 9 (e.g., "how many squares are on the screen?"), adolescents with autism seemed to show evidence of a smaller subitizing range than nonautistic adolescents, although the groups were not compared statistically [see O'Hearn, Franceroni, Wright, Minshew, & Luna, 2013, for replication and extension]. Although all three studies assess precise enumeration using different methods, none is suggestive of superiorities in the exact system in autism.

It remains possible, however, that estimation of *approximate* number might be enhanced in autism. This skill is typically assessed using psychophysical paradigms whereby participants are briefly shown two patches of dots and asked to report which patch contains more dots [see Fig. 1, and Ansari, 2008, for review]. Some researchers [Mottron, Dawson, Soulières, Hubert, & Burack, 2006; Plaisted, 2001] have proposed that such perceptual operations are generally enhanced in autism, rendering it possible that many autistic individuals might excel at numerosity.

Furthermore, it is assumed that the ability to represent numerosity or *nonsymbolic* numerical magnitude (rather than subitizing) is a prerequisite for the later acquisition of symbolic representations of numerical magnitude [Feigenson et al., 2004]. Consistent with this view, the ability to estimate numerosity correlates strongly with mathematics achievement at different ages [Anobile, Stievano, & Burr, 2013; Gilmore, McCarthy & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Piazza et al., 2010; although see Tibber et al.,

Table 1. Descriptive Statistics and Group Differences for Chronological Age, Measures of Intellectual Functioning, Autistic Symptoms, and Mathematical Achievement

	Group			
	Children with autism ($n = 32$)	Children without autism ($n = 32$)	P-value	Effect size (Cohen's d)
Age (in months)				
M (SD)	123.33 (15.6)	119.09 (12.8)	0.24	0.29
Range	95.4–158.2	93.8–148.6		
Full scale IQ[a]				
M (SD)	106.3 (9.6)	108.22 (12.8)	0.50	0.17
Range	89–128	79–130		
Verbal IQ[a]				
M (SD)	103.09 (10.8)	107.56 (12.0)	0.12	0.32
Range	88–132	87–133		
Performance IQ[a]				
M (SD)	108.41 (12.6)	107.25 (13.4)	0.72	0.09
Range	85–138	76–129		
SCQ[b]				
M (SD)	24.62 (5.4)	3.16 (3.0)	<0.001**	4.89
Range	15–33	1–11		
WOND mathematical reasoning[c]				
M (SD)	100.78 (14.9)	108.94 (15.5)	0.036*	0.54
Range	75–139	83–149		
WOND numerical operations[c]				
M (SD)	94.25 (13.6)	105.9 (16.1)	0.003**	0.78
Range	70–124	73–138		
WOND composite score[c]				
M (SD)	97.09 (15.0)	107.31 (16.6)	0.012*	0.64
Range	73–130	74–147		

Notes: [a]Children's intellectual functioning was measured using the Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 1999), standard scores reported here.

SCQ: Social Communication Questionnaire (Rutter et al., 2003), scores out of 39.

WOND: Wechsler Objective Numerical Dimension (Rust, 1996), standard scores reported here.

$*P < 0.05$; $**P < 0.005$.

2013]. Individual differences in nonsymbolic estimation (numerosity) could, therefore, provide one explanation for the variability in arithmetic abilities in autism.

Another potential source of individual differences in mathematics is *mapping* number to space, asking children to indicate the position of a symbolic digit or a cloud of dots on a "numberline" [Booth & Siegler, 2006]. Younger children tend to overestimate smaller numbers and compress larger numbers at the end of the line, producing estimates that are better fit by a logarithmic than a linear function. Older children and adults tend to produce more accurate, more nearly linear estimates. Importantly, individual differences in children's symbolic estimation, their ability to map numbers onto space is also linked to variation in mathematical achievement [Booth & Siegler, 2006; Piazza et al., 2010; Siegler & Booth, 2004].

The Present Study

The aim of this research was to investigate number estimation in children with autism. First, we assessed cognitively able 8- to 13-year-old children with and without autism on an engaging and developmentally sensitive *nonsymbolic estimation* (numerosity) task. Second, we determined their *symbolic estimation* by measuring their ability to map symbolic representations onto space with two numberline tasks. Finally, we also sought to establish the extent to which individual differences in these skills related to differences in mathematical achievement.

Method

Participants

Thirty-two children with autism (28 boys) and 32 typical children (26 boys), aged between 8 and 13 years, took part (see Table 1). Children were recruited from community contacts and schools in London and surrounding areas. All children with autism had received an independent clinical diagnosis according to DSM-IV criteria [APA, 2000], including autism ($n = 25$) and Asperger syndrome ($n = 7$), and met thresholds for an autism spectrum disorder on the Social Communication Questionnaire [SCQ; Rutter, Bailey, & Lord, 2003] and the Autism Diagnostic Observational Scale–Generic [ADOS-G; Lord, Rutter, DiLavore, & Risi, 1999]. Four

Table 2. Table Shows the Five Experimental Conditions on the Numerosity Task, Including the Conditional Exponents and the Relationship Between Both Area and Density to Numerosity

Condition	Inverse AREA	Constant area	Minimal increase	Constant density	Inverse density
CE	-0.5	0.0	0.5	1.0	1.5
Area	$\propto (-0.5)$N	Constant	$\propto (0.5)$N	$\propto$ N	$\propto (1.5)$N
Density	$\propto (1.5)$N	$\propto$ N	$\propto (0.5)$N	Constant	$\propto (-0.5)$N

additional autistic children and threee typical children were assessed but were removed from all analyses for poorly-fitting functions (i.e., those with flat or negative slope values) on the numerosity task (see below).

There were no group differences on chronological age, $F(1, 62) = 1.42$, $P = 0.24$, or full-scale IQ, $F(1, 62) = 0.47$, $P = 0.50$, as measured by the Wechsler Abbreviated Scales of Intelligence [WASI; Wechsler, 1999] (see Table 1). All children were considered "cognitively able," achieving FSIQ scores greater than 70.

Measures

Standardized measure of mathematical achievement. Mathematical achievement was measured using the Wechsler Objective Numerical Dimension [WOND; Rust, 1996], which comprises two subtests. The first subtest, *Mathematical Reasoning*, consisted of verbal-based numerical and geometric problems. The second subtest, *Numerical Operations*, consisted of paper-and-pencil arithmetic operations of increasing difficulty. Standard guidelines for administration and scoring were followed. The test yields standard scores ($M = 100$; SD = 15) for each subtest and an overall *Composite Score*.

Nonsymbolic estimation. In the numerosity discrimination task, children were introduced to two animated characters in competition to see who could win the most marbles (Fig. 1). Children were asked to help the friendly character, Zando. They would see two sets of "marbles" (dots) and their job was to touch the side of the screen with more "marbles." Children received no feedback on accuracy.

There were two blocks of 100 trials (200 trials total) with each trial initiated by the experimenter when children were attending to the central fixation point. The stimuli comprised two patches of high-contrast dots, half of which were bright, half dark, to ensure that mean luminance did not vary with numerosity. The standard patch contained 48 dots within a circle of 10° diameter (230 pixels) covering 7.3% of the defined area. Stimuli were displayed simultaneously for 500 ms at 10 degrees left and right of the fixation point with the side of the standard (48 dots) and probe (varying number) stimulus counterbalanced.

The probe stimulus was varied using a QUEST adaptive algorithm [Watson & Pelli, 1983], which adjusts the probe value on a trial-by-trial basis by using the previous responses of the participant to estimate the point of subjective equality (PSE), where both patches are perceived as being equal (i.e., responses to the probe are 50% "fewer" and 50% "more"). Given that we were interested in both the PSE and the slope of the psychometric function (to estimate children's Weber's fractions), we perturbed the QUEST estimate by a Gaussian jitter ($\sigma = 0.15$ log units) to ensure better distribution of trials in the most sensitive portion of the psychometric curve (0.25–0.75 range), allowing a more accurate estimate of children's Weber fraction. This also ensured that initially selected probe numbers were easy (with large variances in the probe stimulus relative to the 48 dots standard) while in later trials probe numbers were more targeted toward individual children's performance.

To control the potentially confounding effects of density and area (possible cues for numerosity), five different conditions were run such that density and area could either be constant, positively correlated or inversely correlated with numerosity (see Table 2). This was achieved by using the following formulae,

$$A_P = A_S \times \left(\frac{N_P}{N_S}\right)^{CE}$$

$$D_P = D_S \times \left(\frac{N_P}{N_S}\right)^{1-CE}$$

whereby the current probe number (N_P) and number (N_S), area (A_S), and density (D_S) of the standard, and the conditional exponent (CE; either -0.5, 0, $+0.5$, $+1.0$, or $+1.5$) determined the area (A_P) and density (D_P) values for the probe in each condition. As trials from each condition were randomly presented during the session, numerosity was the only consistently valid cue across conditions; the use of other cues (area, density) would either be uninformative or even hinder performance.

The proportion of "more" trials was plotted against probe numerosity and fit with a cumulative Gaussian function where the median (50% point) provided an estimate of the PSE and the normalized standard deviation gave an estimate of participants' Weber fraction. Weber fractions, reflecting the *precision* with which two numerical quantities can be discriminated, were estimated for all five conditions to determine any effects of condition on performance. A mean Weber fraction was also calculated from the key three conditions (constant

Figure 2. Symbolic estimation: the numberline tasks. Children were asked to help a postman deliver presents to a street of houses. They were asked to touch the line (road) at the point that best represented the target number, in this case "house 59." There were 40 trials per numberline (1–100 and 1–1000).

density, equal area and minimal increase) excluding the two inverse parameter catch-trial conditions, to facilitate comparison with previous research. Higher mean Weber fractions reflect poorer nonsymbolic (numerosity) estimation.

Symbolic estimation. Two computer-based numberline tasks, ranging from 1 to 100 and 1 to 1000, were developed following existing pen-and-paper tasks [e.g., Barth & Paladino, 2011; Siegler & Booth, 2004]. In both tasks, children were introduced to a postman character who had mislaid his glasses (Fig. 2) and were asked to help him deliver presents to houses along a single "street" (line). To begin, children were asked to indicate where house 1 and house 100 (or 1000) were located on the "street." Once they correctly identified the endpoints, these remained on-screen throughout. On each trial, children were shown a picture of a present and asked "Where is house number X?" Children indicated their response by touching the touch-sensitive screen with their finger where they thought a particular number (e.g., "35") was located along the line.

Each numberline task comprised 40 trials, the numbers of which were randomly pre-selected and spanned the whole numberline. The same numbers were used for the 1–1000 task as in the 1–100 task, simply multiplied by 10. Each child received the same numbers, presented in a randomized order. Children received no feedback regarding accuracy.

The dependent variable of interest was the total root-mean-square error, that is, the square root of the average squared difference from the location they selected to the actual location of the number. Greater errors reflect poorer symbolic estimation performance.

General Procedure

Children were tested individually in a quiet room either at school or at the Institute. The WASI was always administered first, followed by the first set of number-estimation tasks (the numerosity task and one numberline task), then WOND, and finally the remaining numberline task. The order of presentation of the numberline tasks (1–100 first, 1–1000 first) was counterbalanced across participants.

Stimulus presentation was controlled by Matlab R2010a [The Mathworks Ltd.] on a Dell 3 Precision M6500 laptop with a 17″ screen. Children were seated approximately 57 cm from the screen.

Results

Group Differences

Mathematical achievement. Children with autism performed significantly worse than typical children on both WOND subtests, Mathematical Reasoning, $F(1,63) = 4.62$, $P = 0.04$, $\eta_p^2 = .07$, and Numerical Operations, $F(1, 63) = 9.76$, $P = 0.003$, $\eta_p^2 = 0.14$, and on the overall Composite Score, $F(1,62) = 6.66$, $P = 0.01$, $\eta_p^2 = 0.10$ (see Table 1).

Following Jones et al. [2009], we further examined whether children's scores on the WOND subtests were significantly lower (or higher) than expected given their intellectual ability (WASI FSIQ performance)—that is, whether their mathematical achievement "dips" below

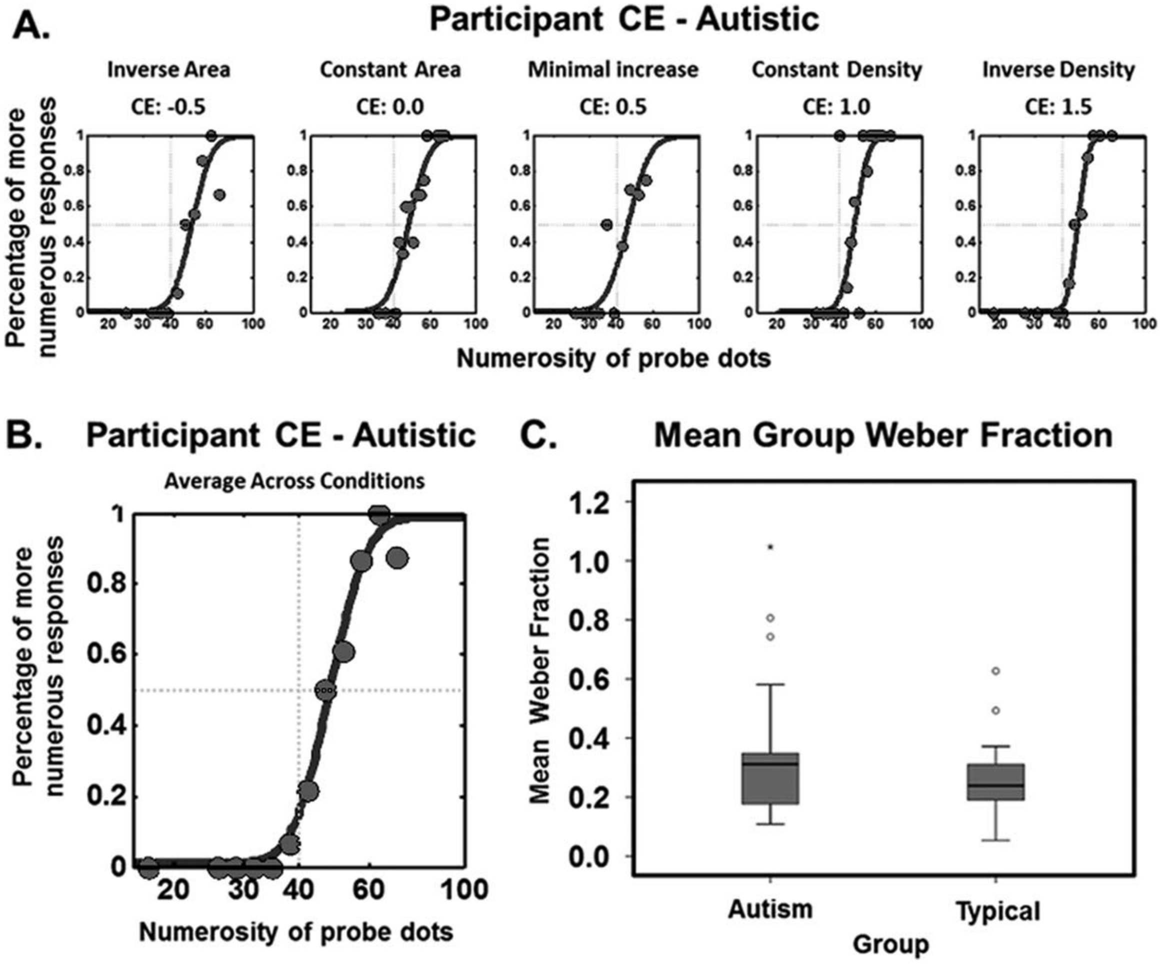

Figure 3. Numerosity task performance. Panel A: Example psychometric functions from one participant with autism for the five different conditions. Panel B: An example function of the same participant produced by averaging his performance on the 0.0, 0.5, and 1.0 conditions (excluding the inverse conditions). Panel C: Box plots showing performance on the numerosity task for the groups of children with autism and typical children. Upper and lower ends of boxes represent 75th and 25th percentiles, respectively. "Whiskers" attached to the boxes extend out to include 100% of the data, with the exception of outliers (<3 SDs), represented by open circles. The median of the distribution is depicted by a solid black line bisecting the box.

(or "peaks" above) their ability. We calculated the discrepancy between their ability (WASI FSIQ scores) and mathematical achievement (WOND subtest scores) using the tables provided in the WOND manual (1996, p. 114, Table C.5). The cut-off points for discrepancy scores for Mathematical Reasoning (MR) Numerical Operations (NO) was 12 and 15 points, respectively. There were significantly more autistic children (MR: $n = 11$; NO: $n = 13$) whose mathematical achievement dipped *below* their intellectual ability compared to typical children (MR: $n = 4$; NO: $n = 1$), $\chi^2 = 13.29$, $P < 0.001$ and $\chi^2 = 3.56$, $P = 0.05$, respectively. Few children with autism (MR: $n = 1$; NO: $n = 1$) and typical children (MR: $n = 4$; NO: $n = 1$) showed an arithmetic peak.

Nonsymbolic estimation. Four autistic children did not complete the numerosity task due to computer faults (leaving $n = 28$). Inspection of the psychometric curves revealed good fits to the data (R^2 values for all curves exceeded 0.9). Data screening identified one outlier, a particularly high Weber fraction (child with autism). This outlier was retained to increase statistical power but the outlying score was replaced with a threshold value corresponding to +2.5 SDs [Tabachnick & Fidell, 2007]. Participants' data were then bootstrapped with random selection 500 times to generate standard errors for each participant's fit. Figure 3 shows the psychometric curves for each condition (A), and across conditions (B) for one example child with autism. The box-and-whisker plot (Fig. 3C) shows the distribution of autistic and typical participants' mean Weber fractions.

A one-way ANOVA on children's mean Weber fractions revealed a significant main effect of group, $F(1, 58) = 7.93$, $P = 0.007$, $\eta_p^2 = 0.12$. Autistic children

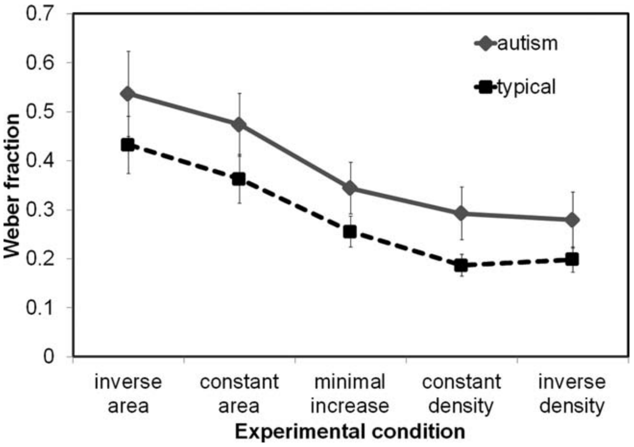

Figure 4. The average Weber fraction for the autism and typical groups across the five experimental conditions of the numerosity task. Error bars reflect ± SEM.

obtained higher Weber fractions ($n = 28$; $M = 0.48$; $SD = 0.38$; i.e., worse performance), on average, than typical children ($n = 32$; $M = 0.27$; $SD = 0.13$), that is, they needed a larger difference between the numbers to discriminate accurately numerosity. Inspection of Figure 3C suggests that the distribution of scores nevertheless overlapped considerably.

To determine whether the groups of children relied on the different (area, density) cues to a similar degree, we conducted a mixed ANOVA on children's Weber fractions using all five conditions as a repeated-measures factor in addition to group (Fig. 4). There was a main effect of condition, $F(4, 232) = 11.55$, $P < 0.001$, $\eta_p^2 = 0.17$. Children performed worse on conditions in which the area cue was either unavailable or inversely correlated with numerosity [see Hurewitz, Gelman, & Schnitzer, 2006]. There was also a main effect of group, $F(1, 58) = 4.94$, $P = 0.03$, $\eta_p^2 = 0.08$, with autistic children performing worse overall than typical children. Critically, there was no significant condition × group interaction, $F < 1$. Children across groups were either helped or hindered by area/density cues to a similar extent.

Finally, analyses on children's PSEs showed no group differences in performance biases (e.g., between responding to the left patch of dots and responding to the right), $F(1, 58) = 1.80$, $P = 0.19$.

Numberline tasks. Figure 5 shows example results from two typical children and two children with autism (one low-performing and one high-performing in each group) on the 1–100 and 1–1000 numberlines. For both tasks, group differences in total error approached significance for the 1–100 numberline (autism: $n = 32$; $M = 10.36$; $SD = 4.70$; typical: $n = 32$; $M = 8.33$; $SD = 3.62$), $F(1, 62) = 3.75$, $P = 0.057$, $\eta_p^2 = 0.06$, and 1–1000 numberline tasks (autism: $n = 32$; $M = 13.36$; $SD = 7.56$; typical: $n = 32$; $M = 10.15$; $SD = 7.11$), $F(1, 62) = 3.36$, $P = 0.07$, $\eta_p^2 = 0.05$.

Figure 6 shows the numberline data, averaged across participants within each group. For both the 1–100 and 1–1000 numberlines (see Figs. 6A and 6B, respectively), there appears to be no significant systematic shifts toward any nonlinear representation for either group of children, with participant errors at the group level distributed relatively uniformly.

Relationship Between Numerical-Estimation Tasks

Initial correlational analyses between children's performance on numerical estimation tasks and developmental variables (age and ability) showed significant negative correlations between age and 1–1000 number line performance for children with autism, $r(31) = -0.54$, $P = 0.002$, and typical children, $r(31) = -0.43$, $P = 0.015$, but not with other variables (all $ps > 0.16$). There were also significant positive correlations between ability (full-scale raw scores) and nonsymbolic (numerosity) and symbolic (numberline) estimation in both groups (all $ps < 0.05$). The potentially confounding effects of age and ability were, therefore, partialled out of the relationships between individual number-estimation tasks.

In typical children, performance on the 1–100 and 1–1000 numberline tasks were significantly intercorrelated, $r(31) = 0.47$, $P = 0.007$ (Fig. 7). Also, numerosity performance was significantly positively related to 1–100, $r(31) = 0.37$, $P = 0.04$, and 1–1000 numberline tasks, $r(27) = 0.36$, $P = 0.04$; lower Weber fractions were associated with more accurate numberline mapping. When age and ability were controlled for, only correlations between performance on the 1–100 and 1–1000 numberlines, $r(27) = 0.39$, $P = 0.03$, and the 1–100 numberline and numerosity performance, $r(27) = 0.36$, $P = 0.048$, remained significant.

There was a similar pattern of performance in children with autism. There were significant relationships between errors on the 1–100 and 1–1000 numberlines, $r(31) = 0.54$, $P = 0.002$ (Fig. 7) and between children's numerosity and 1–1000 numberline performance, $r(27) = 0.40$, $P = 0.04$, but not between numerosity and 1–100 numberline performance, $r(27) = 0.21$, $P = 0.28$. The correlations between performance on the 1–100 and 1–1000 numberlines, $r(23) = 0.55$, $P = 0.004$, and the 1–1000 numberline and numerosity, $r(23) = 0.41$, $P = 0.04$, remained significant once variation in age and ability were partialled out.

Relationship Between Mathematical Achievement and Other Variables

Table 3 reports the raw and partial correlations between mathematical achievement variables (WOND subtest scores and overall Composite Score) and numerical estimation

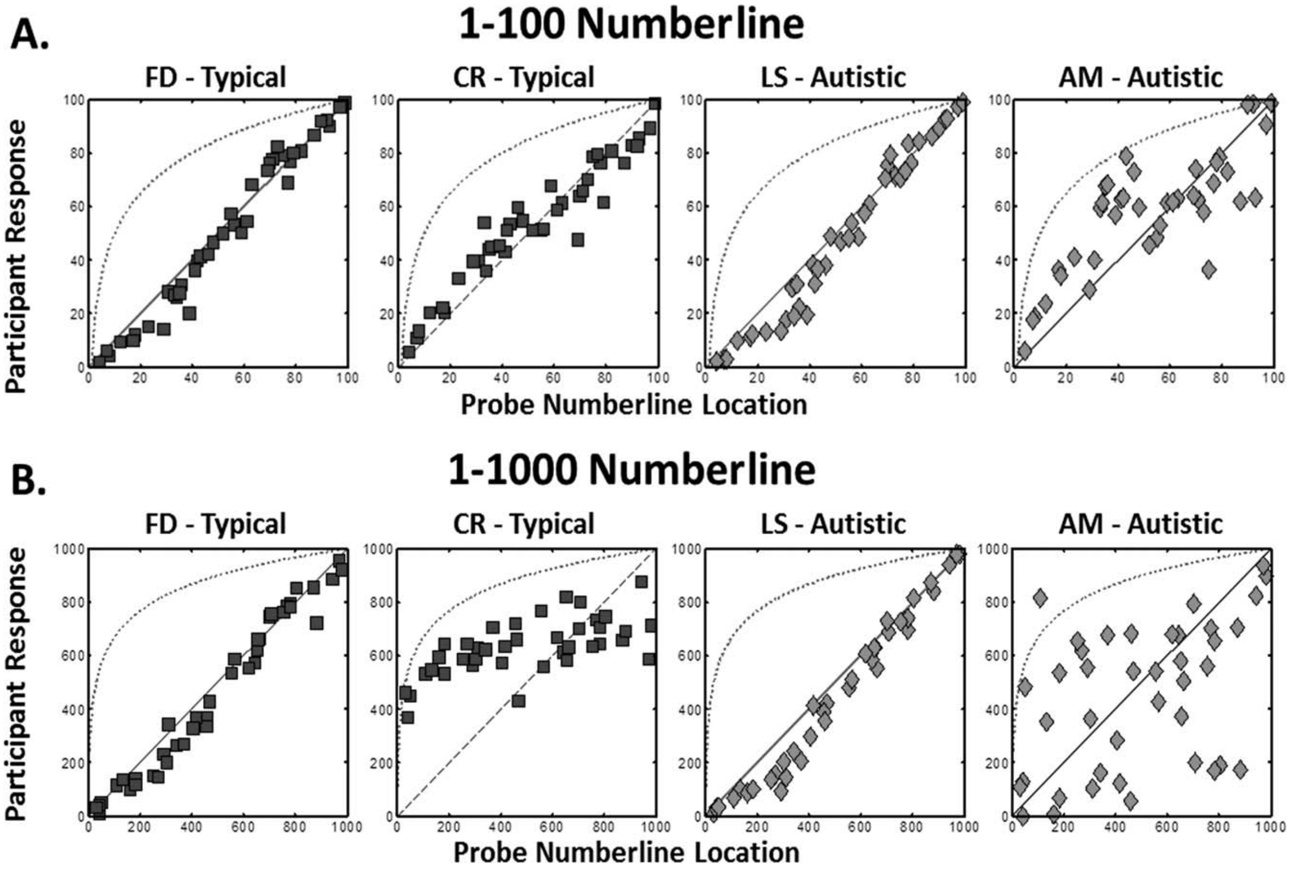

Figure 5. Numberline performance. Example numberline plots for the 1–100 (Panel A) and 1–1000 (Panel B) numberline range showing the distribution of responses for two typical children and two autistic children. The probe number that children were required to indicate is plotted on the *x*-axis, their responses on the *y*-axis. Each row consists of one typical high performing child (FD), one typical low-performing child (CR), a high-performing child with autism (LS), and a low-performing child with autism (AM). The solid line indicates the equality line (linear) while a dashed line reflects a logarithmic distribution. Although there is some evidence amongst participants for an overestimation or compression of the high end of the numberline (particularly in the 1–1000 numberline task), this is not sufficiently consistent across participants to be indicative of logarithmic encoding.

(children's mean Weber fractions and total error on the 1–100 and 1–1000 numberlines) in each group separately.

For typical children, age and intellectual ability scores were highly positively related to scores on the WOND. Furthermore, performance on both numberline tasks was significantly negatively correlated with mathematical achievement; fewer errors on the numberlines were related to higher WOND scores. None of these correlations, however, survived when differences in age and ability were partialled out.

For autistic children, age (but not intellectual ability) was positively related to scores on the WOND. Furthermore, individual differences in performance on the 1–100 and 1–1000 numberline tasks were significantly related to mathematical achievement. Unlike typical children, partial correlations revealed that performance on both numberline tasks remained significantly negatively correlated with scores on all mathematical achievement variables; fewer errors on the numberline tasks were related to higher WOND scores.

Regression analysis. A hierarchical regression analysis was performed to determine the extent to which distinct number-estimation skills (numerosity and numberline mapping) uniquely predicted mathematical achievement (as indexed by WOND Composite scores[2]) for children with and without autism. Including all children in the same model increased statistical power and allowed us to test for an interaction between diagnostic status and number-estimation variables.

As age and ability were related to mathematical achievement, individual differences in these variables were accounted for by entering age, ability (full-scale

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[2] Examination of the correlations reported in Table 3 showed that the patterns of relationships between performance on the number estimation tasks and the subtests, Mathematical Reasoning and Numerical Operations, were very similar to each other and to the overall Composite score. For this reason, and to avoid too many analyses, WOND composite scores rather than individual subtest scores were used for the regression analysis. Supplementary regression analyses on the individual subtests also revealed similar results to those presented in the text (available on request from the authors).

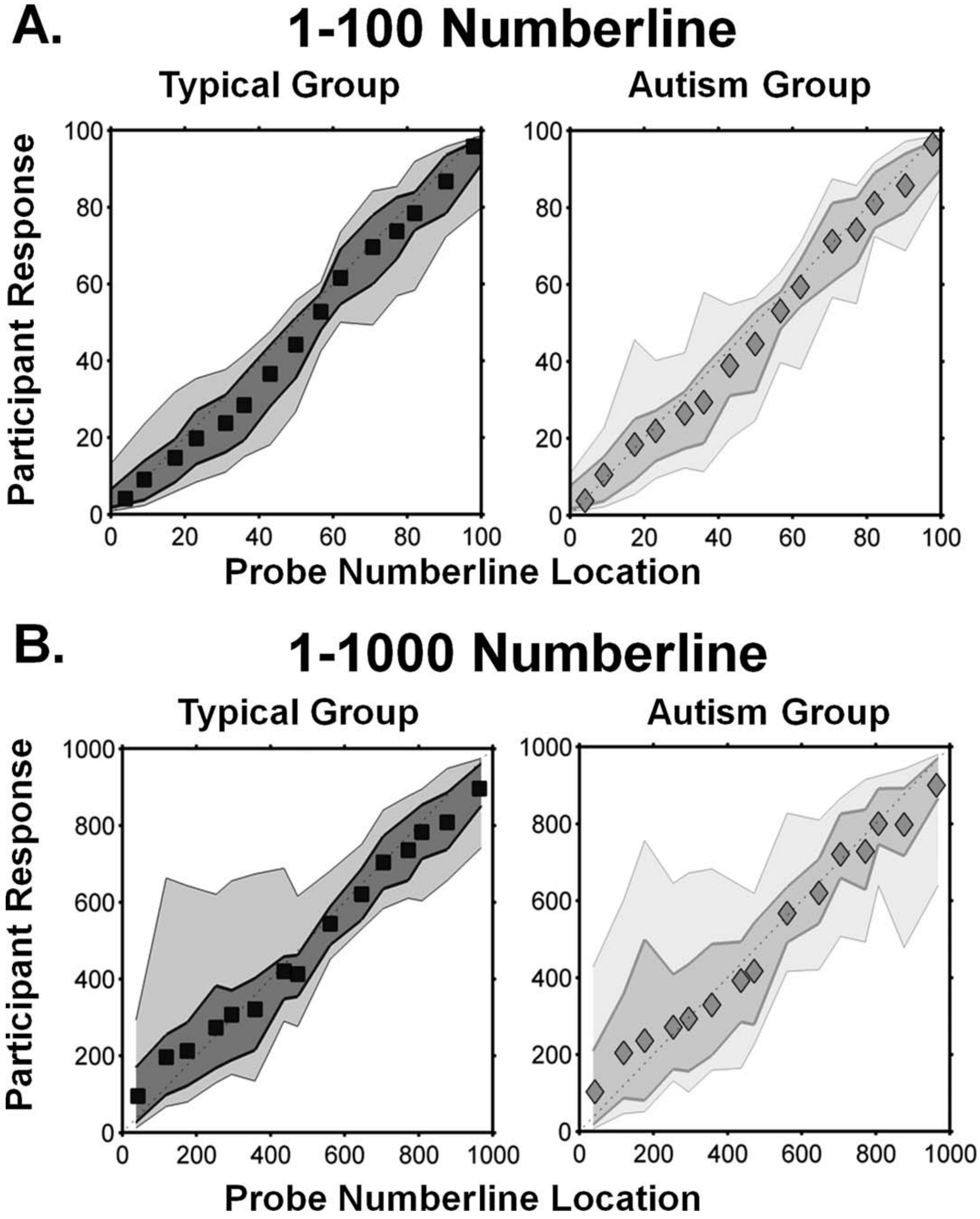

Figure 6. Plots of the numberline response patterns for the 1–100 (Panel A) and 1–1000 (Panel B) numberline range showing the distribution of responses for the typical group and autism group. The diamond symbols indicate the mean response of the typical group while the square symbols indicate the mean response of the autism group. The darker innermost region shows the bounds of one standard deviation quantile range (15.9–86.1% quantiles) while the lighter outermost region shows the 5–95% quantile range of all participant responses.

raw) scores, and diagnostic status in the first step of the regression model. The additional—and potentially unique—contribution of number-estimation variables was then tested by entering them into subsequent steps of the regression equation as well as the interaction terms for each variable.

When age, ability, and diagnostic status were entered simultaneously as predictors of children's WOND Composite Scores, these variables accounted for 64% of the variance, $F(3,56) = 32.94$, $P < 0.001$. Numerosity and both numberline variables were then entered stepwise into the regression, together with their respective interaction terms. Only performance on the 1–1000 numberline task explained an additional 4% of the variance in children's WOND scores, $F(1,55) = 6.62$, $P = 0.01$. The negative beta value (Table 4) suggests that better symbolic estimation predicted better mathematical attainment. The final model was significant, $F(4,59) = 28.84$,

$P < 0.001$, $R^2 = 0.68$. None of the interaction terms were significant (all ps > 0.49) suggesting that autistic children showed a similar pattern of relations among number-estimation skills and mathematical achievement as typical children.

Discussion

Our objectives were threefold: to compare (1) nonsymbolic estimation (numerosity discrimination) and (2) numberline mapping in school-age children with and without autism using child-friendly and developmentally-sensitive tasks; and (3) to establish whether individual differences in nonsymbolic and symbolic estimation skills are related to variation in mathematical achievement.

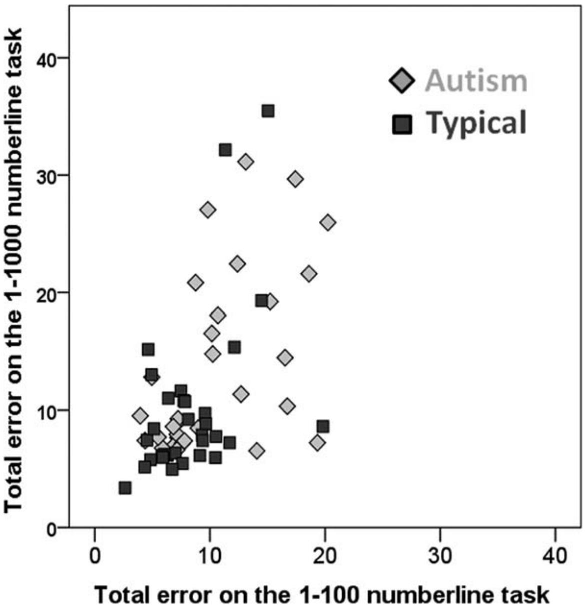

Figure 7. Scatterplot showing the relationship between children's performance on the 1–100 and 1–1000 numberline tasks.

Mathematical Achievement in Autism

Contrary to popular conceptions of superior mathematical skills in autism [see Baron-Cohen, 2002], we found that autistic children were, on average, significantly *worse* in their formal mathematical achievement, as measured by the individual subtests and Composite Score of the WOND. This group difference, however, was not accompanied by greater variability in the scores of children with autism relative to typical children (reflected by a similar range), as previous studies have reported [Jones et al., 2009; Mayes & Calhoun, 2003a]. Unlike the current study, which included only cognitively able children with autism, previous studies have also included autistic individuals with additional intellectual disabilities [Jones et al., 2009: IQ range = 50–119; Mayes & Calhoun, 2003a: IQ range = 14–143], which might well have contributed to the reported heterogeneity in these studies. Indeed, when Mayes and

Table 4. Summary of Hierarchical Regression Analyses Predicting Mathematical Achievement (WOND Composite Scores) (Final Model)

Variable	*B*	*SE B*	*ß*	R^2 or ΔR^2
Step 1				0.64**
Age	0.41	0.07	0.48**	
Ability[a]	0.25	0.04	0.50**	
Diagnostic status	3.46	2.01	0.14	
Step 2				0.04*
1–1000 numberline[b]	−0.37	0.14	−0.22*	

Notes. [a]Indexed by WASI raw (full-scale) ability scores.
[b]Indexed by total error on the task
*Significant at $P < 0.05$; **Significant at $P < 0.01$.

Table 3. Pearson Correlations for Developmental (Age and Ability), Number-Estimation, and Mathematical Achievement Variables in Children With Autism and Typical Children Separately

		WOND mathematical reasoning	WOND numerical operations	WOND composite score
Autism	Age	0.41*	0.35*	0.41*
	Ability[a]	0.20	0.24	0.23
	Numerosity	−0.19(−0.15)	−0.02(0.05)	−0.13(−0.07)
	1–100 numberline	−0.38*(−0.44*)	−0.39*(−0.34)	−0.40*(−0.43*)
	1–1000 numberline	−0.59**(−0.49**)	−0.47**(−0.29)	−0.57**(−0.44*)
Typical	Age	0.69**	0.68**	0.70**
	Ability	0.51**	0.56**	0.55**
	Numerosity	−0.18(−0.12)	−0.08(−0.12)	−0.13(−0.02)
	1–100 numberline	−0.39*(−0.03)	−0.33(0.10)	−0.37*(0.03)
	1–1000 numberline	−0.47**(−0.16)	−0.52**(−0.22)	−0.51**(−0.21)

Notes. Partial correlations adjusting for age and intellectual ability are shown in parentheses as indexed by raw (full-scale ability) scores on the WASI (Wechsler, 1999).
*Significant at the 0.05 level (2-tailed); **Significant at the 0.01 level (2-tailed).

Calhoun [2003b] divided their autism group into those children with full-scale IQ scores above ($n = 42$) and below 80 ($n = 21$), they found that for those with an IQ above 80, their standard scores on the mathematical achievement test showed a similar degree of variability (M score = 96; SD = 16; range = 61–135) as those reported herein.

When we examined further mathematical skills within each group, we found a significant proportion of children with autism whose mathematical achievement was incommensurate with their intellectual ability [i.e., they showed an arithmetic "dip"; cf., Jones et al., 2009]. Such a discrepancy suggests that these children may have an additional, specific learning disability in mathematics. While developmental dyscalculia reportedly affects between 3% and 6% of the general population [for review see Shalev, Auerbach, Manor, & Gross-Tsur, 2000], we found that arithmetic ability dipped significantly below intellectual functioning—indicative of dyscalculia—in 32–40% of our sample of children of autism. These figures are noteworthy, if not somewhat troubling, particularly given that the public notion of mathematical ability in autism is as a relative strength, not difficulty. These results should be replicated in a larger (ideally population-based) sample of school-age children but they nevertheless suggest that practitioners and parents should be attentive to potentially specific difficulties in mathematics in otherwise intellectually-able children with autism.

Number Estimation in Autism

Our study also examined the integrity of number-estimation processes in autism, including nonsymbolic and symbolic estimation, both of which are thought to be responsible, at least in part, for developments in mathematical skills. Our sample of autistic children performed worse than typical children on all three number-estimation tasks: on average, they showed poorer nonsymbolic estimation, as indexed by numerosity discrimination, and performed marginally worse on symbolic estimation, as measured by numberline mapping on the 1–100, and more difficult, 1–1000 tasks.

To our knowledge, this is the first study to demonstrate that autistic children may have difficulties with symbolic estimation, specifically mapping numerical representations onto space. Importantly, it is not the nature of these representations that appears to be different in autism (see Figs. 5 and 6). Children with autism seem to be using linear (as opposed to logarithmic) sequencing of symbolic numbers along the numberline—just like typical children—but with less accuracy. Caution is warranted, however, given that these differences only approached significance.

Only one other study has investigated nonsymbolic (numerosity) perception in autism [Meaux, Taylor, Pang, Vara, & Batty, 2014]. In the context of a magnetoencephalography (MEG) study, autistic ($n = 14$) and typical ($n = 14$) adults were asked to estimate verbally the number of dots in stimuli arranged in either a nonmeaningful or a meaningful (e.g., animal) way, presented for 1,000 ms each. Unlike the current results, the authors found no specific impairment in numerical estimation in their autistic adults. The paradigm used by Meaux et al. to assess numerical estimation, however, is very different to the more sensitive psychophysical procedure used herein. It is possible that adults with autism might show difficulties on a task requiring them to make very subtle judgments in differences in numerosity. Or, alternatively, that numerosity difficulties in autistic individuals abate with age.

Overall, our findings suggest that children with autism, on average, show reduced sensitivity to differences between ensembles of numerosities (nonsymbolic estimation) and are also less able to map between symbolic and spatial representations of number using the numberline (symbolic estimation)—results that are in direct contrast to the predictions of some theoretical accounts. Baron-Cohen [2002] has suggested that heightened systemizing could explain some autistic individuals' prodigious talents in mathematical calculation, considerable achievement in mathematics "in the high-functioning cases" (p. 252) and the preponderance of males in mathematics, physics, and engineering disciplines. The fact that we show no general facility in mathematics or number-estimation capacities in our cognitively-able children is problematic for this account. Interestingly, a recent study that more explicitly measured systemizing in a general population study failed to find a link between adults' scores on the Systemizing Quotient and the ability to solve mathematical problems [Morsanyi, Primi, Handley, Chiesi, & Galli, 2012]. Together, these findings speak against exceptional number-estimation skills as a general rule in those with autism.

What then might be driving autistic children's *difficulties* in numerical estimation? We tested nonsymbolic estimation by showing children two patches of dots and asking them to identify which patch contained more dots. The brief presentation time meant that it was impossible to count the dots one-by-one. Instead, children needed to take a "snapshot"—or global picture—of the display to make their judgment. One prominent account has suggested that children with autism show a lesser tendency to process global information [Frith & Happé, 1994], which could account for their apparent difficulty on this task. Problems in processing global information are not consistently found in children with autism, however [e.g., see Simmons et al., 2009]. Furthermore, it is less clear, however, how such a reduced tendency might impact on the numberline task, where the child must translate between symbolic representations and spatial representations of number.

An alternative possibility is that autistic children's difficulties in number are attributable to atypicalities in a singular magnitude processing system [Walsh, 2003]. The ability to make magnitude judgments not only of numerosity (number estimation) but also of time (duration magnitude estimation) has been proposed to be processed by a cross-domain magnitude comparator (i.e., performing judgments of "how much"). Difficulties in temporal processing have been reported in autism [e.g., Allman, DeLeon, & Wearden, 2011]. It is, therefore, possible that atypicalities in time and numerical estimation are driven by underlying problems in a more generalized magnitude estimation system [see Allman, Pelphrey, & Meck, 2012]. Future research should test this possibility by examining perception of time and number in the same children with autism.

Links Between Number Estimation and Mathematical Achievement

Our study further aimed to determine the extent to which underlying number-estimation processes were related to mathematical achievement in children with and without autism. The numberline tasks required knowledge of the symbolic numerical system and mapping of a mental code onto a visual code, while the other task was nonsymbolic, directly related to perception of numerosity. In typical children, mathematical achievement has been shown to correlate with accuracy in nonsymbolic visual comparison tasks [e.g., Gilmore et al., 2010; Halberda et al., 2008] and (symbolic) estimates on a numberline [e.g., Booth & Siegler, 2006; Siegler & Booth, 2004]. We, therefore, expected that individual differences on both tasks would make independent contributions to predicting mathematical attainment in typical children—and potentially also in autistic children.

In both groups, we found that age and intellectual ability explained a significant—and large—amount of the variance in mathematical skills. The magnitude of this relationship is striking but is not dissimilar to the relationship found between scores on the Wechsler IQ scales and the WOND in standardization samples [see Rust, 1996]. Furthermore, and importantly, numberline mapping (as indexed by children's 1–1000 numberline performance) was uniquely related to their mathematical skills, beyond the variance already accounted for by individual differences in age and ability. That is, children who are older and who have greater intellectual and number-to-space mapping ability also have better mathematical skills. The lack of any interaction effects in the regression model suggests that the nature of the relationships between number estimation and mathematical achievement is similar across groups.

We also found that children's nonsymbolic estimation—their numerosity performance—was unrelated to their mathematical achievement in either group. While some studies have found a relationship between typical children's numerosity performance (Weber fractions) and scores on tests of mathematical achievement [e.g., Anobile et al., 2013; Gilmore et al., 2010; Halberda et al., 2008], others have not [e.g., De Smedt & Gilmore, 2011; Holloway & Ansari, 2009]. Indeed, two studies that have examined nonsymbolic and symbolic estimation in the same children have produced contrasting results [Anobile et al., 2013; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013]. What is important about the current study, however, is that the *same pattern* of findings was present across children with autism *and* typical children suggesting that, for these samples of children, processing symbolic numbers seems to be important for mathematical skills, above and beyond variation in intellectual ability.

It is likely that symbolic number processing is not the only (specific) source of the variability in autistic children's formal arithmetic skills. Indeed, components of executive function have been shown to contribute to mathematical achievement in typical children [see Bull & Lee, 2014]. Given that children with autism show problems in executive control [Pellicano, 2013], the extent to which variation in these domain-general skills relate to mathematical ability over and above that already accounted for by symbolic number processing is an important avenue for future research.

In sum, our findings suggest that cognitively able children with autism, on average, show nonsymbolic and symbolic estimation and mathematical achievement skills that are incommensurate with their age and ability. The specific relationship between number-to-space mapping and mathematical achievement in children with autism suggests that they are using their symbolic number-estimation skills in the service of formal mathematical achievement, similar to typical children. Importantly, not all children with autism show difficulties with mathematics or number estimation. But future research should determine the extent of some autistic children's difficulties with regards to number [e.g., by examining their symbolic arithmetic knowledge; Gilmore, McCarthy, & Spelke, 2007] and undertake longitudinal and training studies to pinpoint the precise factors that contribute to the potentially delayed development of autistic children's formal, symbolic mathematical knowledge.

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